Chapter 6

Quadrilaterals

Section 7
Areas of Triangles and Quadrilaterals

GOAL 1: Using Area Formulas

You can use the postulates below to prove several area theorems.

AREA POSTULATES

POSTULATE 22 ***Area of a Square Postulate***

The area of a square is the square of the length of its side, or $A = s^2$.

POSTULATE 23 ***Area Congruence Postulate***

If two polygons are congruent, then they have the same area.

POSTULATE 24 ***Area Addition Postulate***

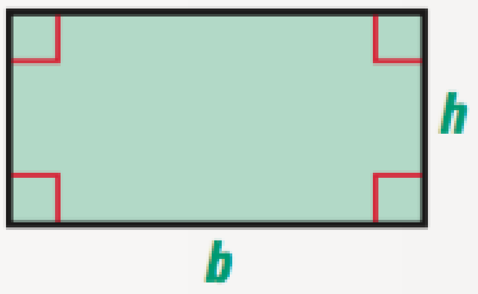
The area of a region is the sum of the areas of its nonoverlapping parts.

AREA THEOREMS

THEOREM 6.20 ***Area of a Rectangle***

The area of a rectangle is the product of its base and height.

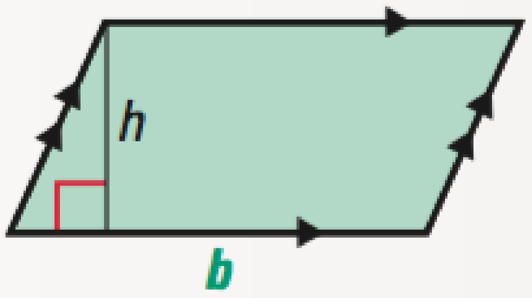
$$A = bh$$



THEOREM 6.21 ***Area of a Parallelogram***

The area of a parallelogram is the product of a base and its corresponding height.

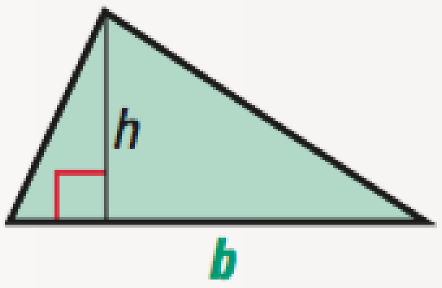
$$A = bh$$



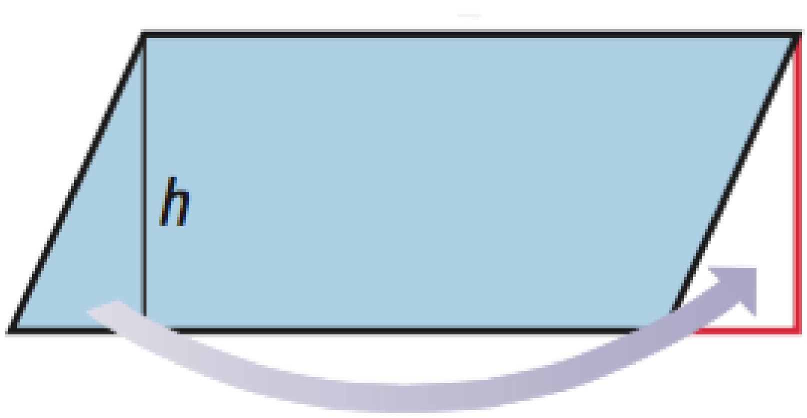
THEOREM 6.22 ***Area of a Triangle***

The area of a triangle is one half the product of a base and its corresponding height.

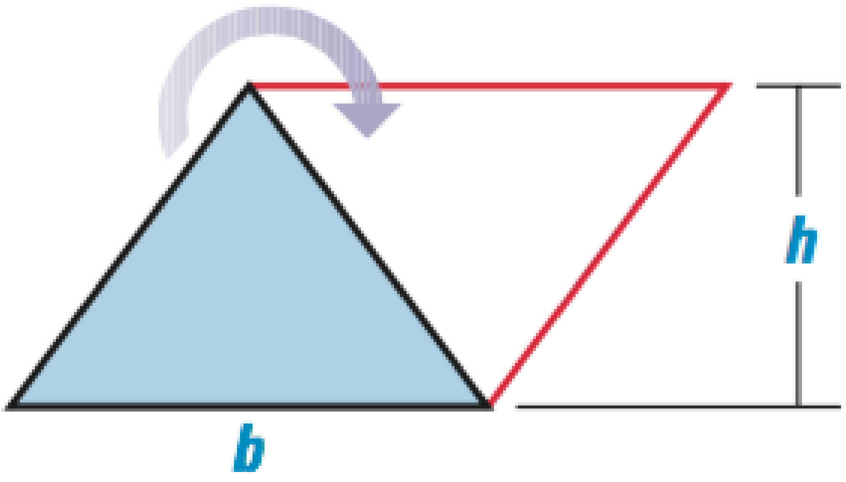
$$A = \frac{1}{2}bh$$



You can justify the area formulas for triangles and parallelograms as follows.



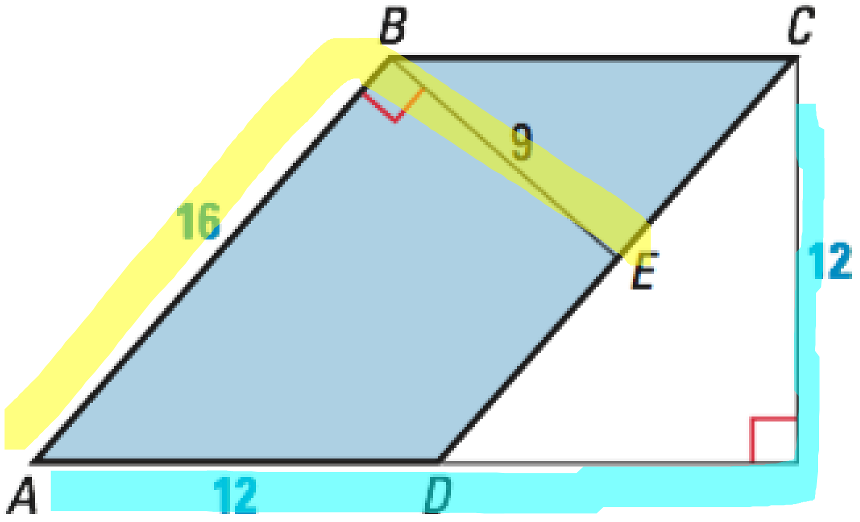
The area of a parallelogram is the area of a rectangle with the same base and height.



The area of a triangle is half the area of a parallelogram with the same base and height.

Example 1: Using the Area Theorems

Find the area of parallelogram ABCD.



$A = bh$

12×12 | 16×9

$144\ u^2$ | $144\ u^2$

Example 2: Finding the Height of a Triangle

Rewrite the formula for the area of a triangle in terms of h. Then use your formula to find the height of a triangle that has an area of 12 and a base length of 6.

$$2 \times A = \cancel{\tfrac{1}{2}} bh \times \cancel{2}$$

$$\frac{2A}{b} = \frac{\cancel{b}h}{\cancel{b}}$$

$$\frac{2A}{b} = h$$

$$\frac{2(12)}{6} = h$$

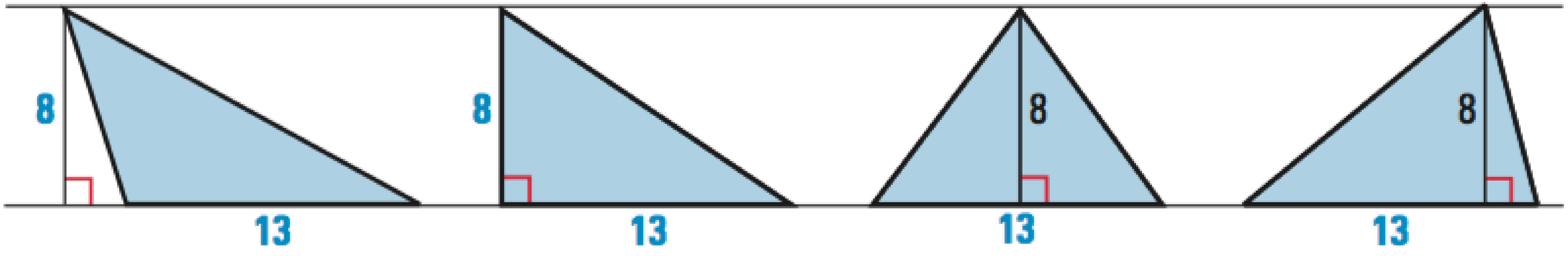
$$4 = h$$

Example 3: Finding the Height of a Triangle

A triangle has an area of 52 square feet and a base of 13 feet. Are all triangles with these dimensions congruent?

**just b/c 2 shapes have the same area, does not mean they're congruent

There are many triangles with these dimensions. Some are shown below.



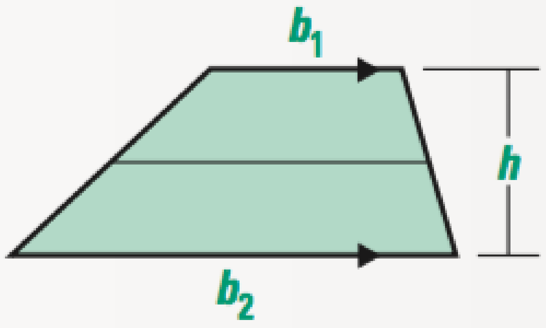
GOAL 2: Areas of Trapezoids, Kites, and Rhombuses

THEOREMS

THEOREM 6.23 ***Area of a Trapezoid***

The area of a trapezoid is one half the product of the height and the sum of the bases.

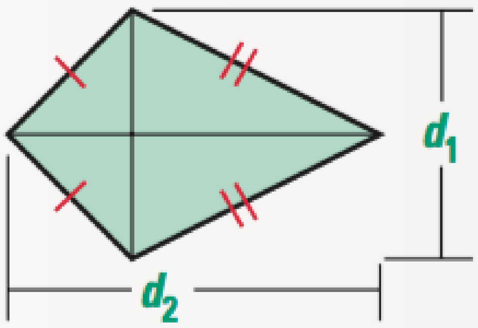
$$A = \frac{1}{2}h(b_1 + b_2)$$



THEOREM 6.24 ***Area of a Kite***

The area of a kite is one half the product of the lengths of its diagonals.

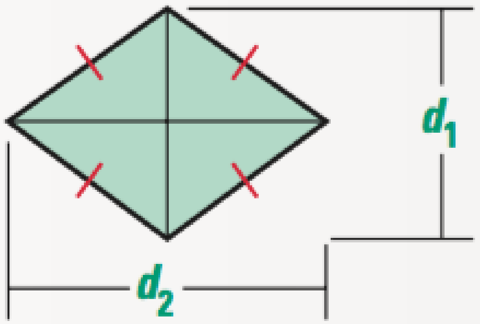
$$A = \frac{1}{2}d_1 d_2$$



THEOREM 6.25 ***Area of a Rhombus***

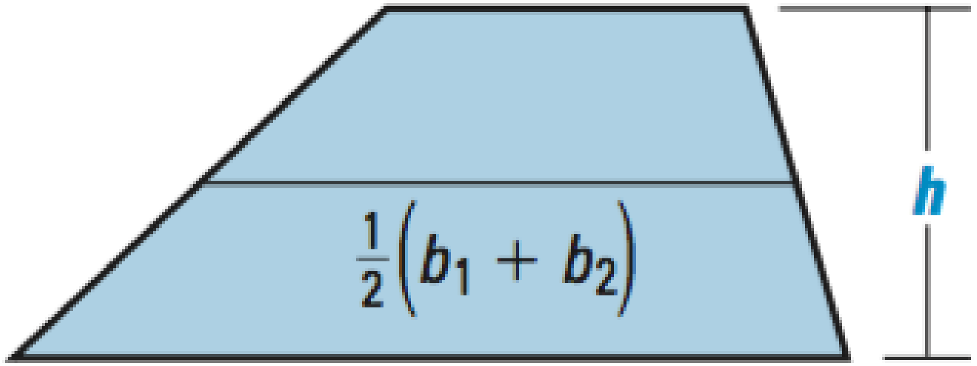
The area of a rhombus is equal to one half the product of the lengths of the diagonals.

$$A = \frac{1}{2}d_1 d_2$$



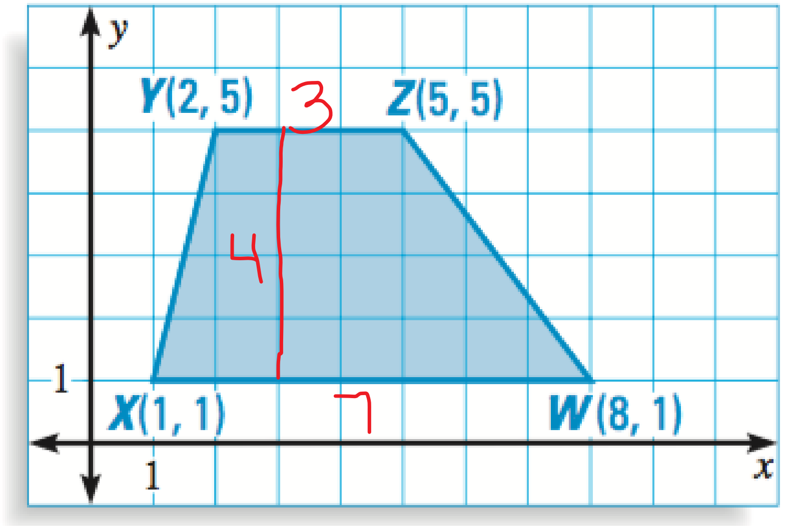
You will justify Theorem 6.23 in Exercises 58 and 59. You may find it easier to remember the formula/theorem the following way.

Area = **Length of Midsegment** • **Height**



Example 4: Finding the Area of a Trapezoid

Find the area of trapezoid WXYZ.



$$A = \frac{1}{2}h(b_1 + b_2)$$

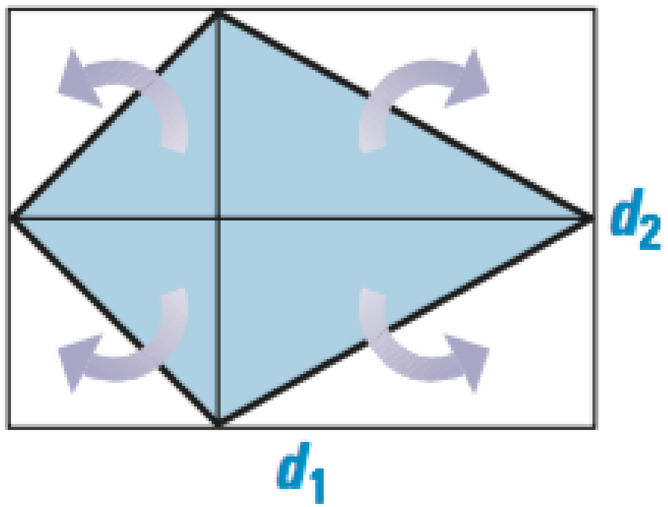
$$\frac{1}{2}(4)(3+7)$$

$$\frac{1}{2}(4)(10)$$

$$20\ u^2$$

The diagram at the right justifies the formulas for the areas of kites and rhombuses.

The diagram shows that the area of a kite is half the area of the rectangle whose length and width are the lengths of the diagonals of the kite. The same is true for a rhombus.



$$A = \frac{1}{2}d_1 d_2$$

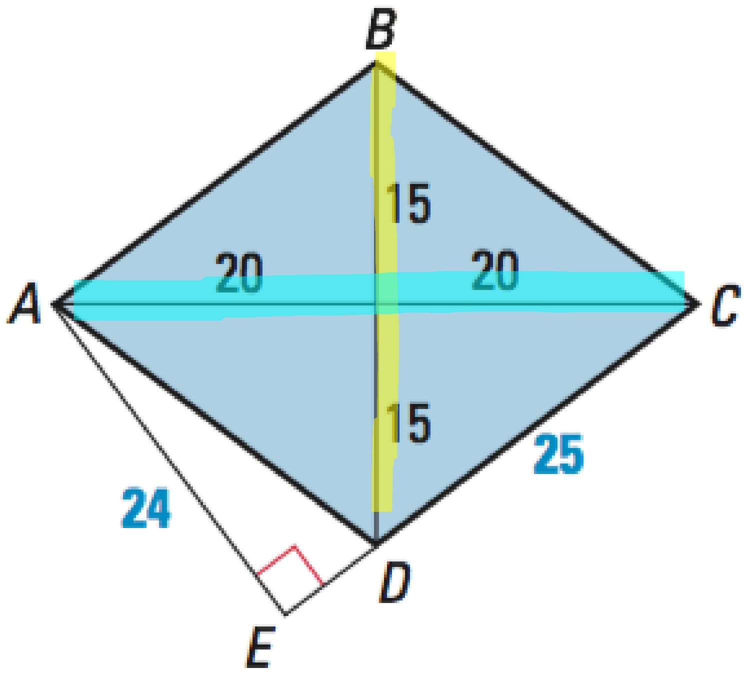
Example 5: Finding the Area of a Rhombus

Use the information given in the diagram to find the area of rhombus ABCD.

$$\frac{1}{2}d_1 d_2$$

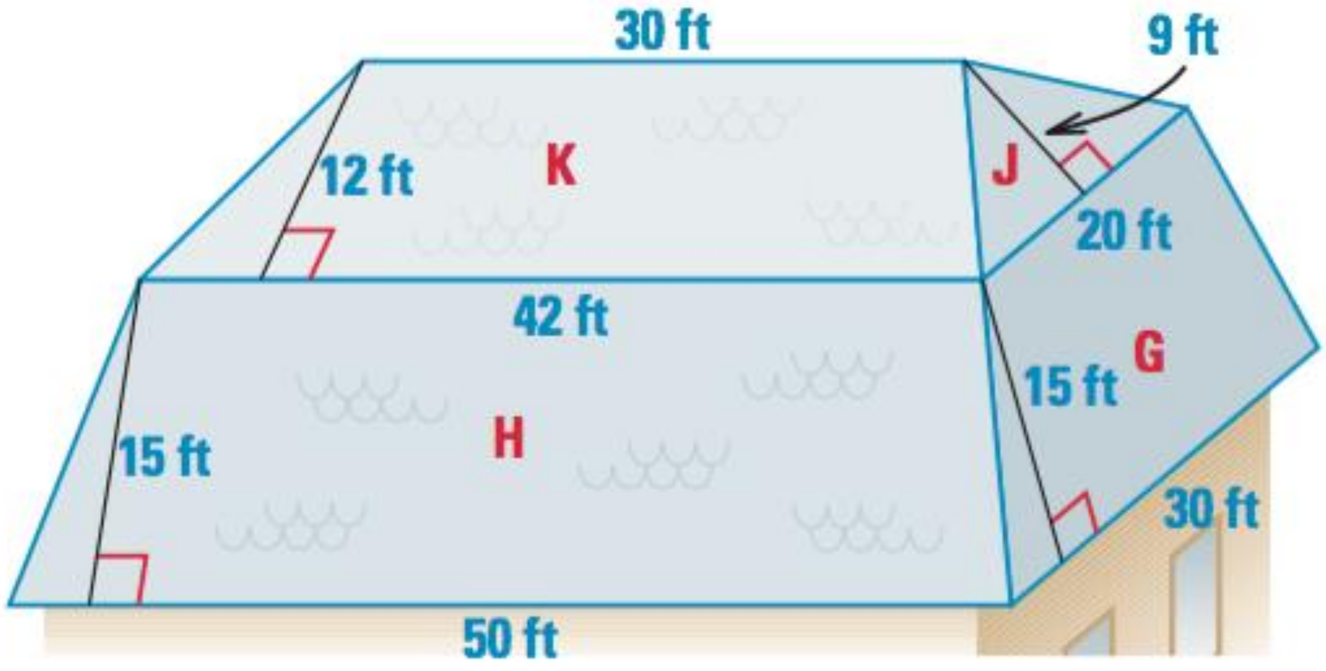
$$\frac{1}{2}(30)(40)$$

$$600\ u^2$$



Example 6: Finding Areas

Find the area of the roof. G, H, and K are trapezoids and J is a triangle. The hidden back and left sides of the roof are the same as the front and right sides.



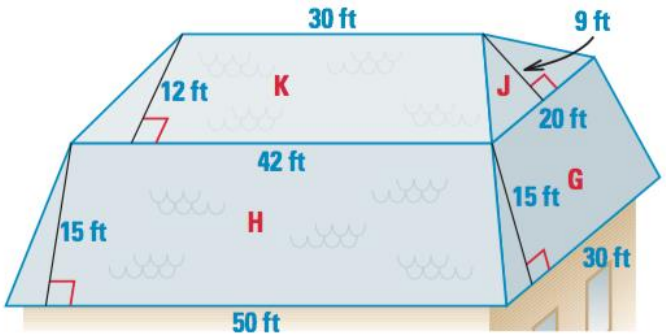
G: $\frac{1}{2}(15)(20+30) = 375 \times 2 = 750$

H: $\frac{1}{2}(15)(42+50) = 690 \times 2 = 1380$

K: $\frac{1}{2}(12)(30+42) = 432 \times 2 = 864$

J: $\frac{1}{2}(20)(9) = 90 \times 2 = 180$

Total: 750 + 1380 + 864 + 180 = 3174 square feet



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